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A Simplified Method for Aseismic Design of Self-Supporting Latticed Telecommunication Towers

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ABSTRACT

The authors propose a simplified static method to evaluate the member force envelope in self-supporting latticed telecommunication towers subjected to horizontal earthquake effects. The method is based on modal superposition. Equivalent horizontal inertia loads are calculated using an acceleration profile based on the lowest three flexural modes of vibration of the tower. Closed form expressions for the acceleration profile are suggested based on the solution of the tapered cantilever of rectangular cross section subjected to harmonic base motion, taking into account the effects of shear deformations. The method is calibrated with results obtained from dynamic analyses of existing latticed towers subjected to several horizontal accelerograms with various peak ground acceleration-to-velocity (A/V) ratios. A simplified bi-linear acceleration profile is suggested. Detailed results are presented for a 90-meter tall three-legged tower and it is found that the proposed static method agrees with dynamic analysis for the accelerograms with medium A/V ratio.

INTRODUCTION

Aseismic design checks are now routinely done in many constructed facilities, and they may be required for special telecommunication towers located in high-risk seismic areas. In order to provide some guidance to Canadian tower designers, the 1994 edition of *CAN/CSA-S37 Antennas, Towers and Antenna-Supporting Structures* devotes a new appendix to the general topic of seismic analysis of towers. Recommendations for seismic analysis of self-supporting towers are limited to detailed modal superposition with base motion input compatible with the seismicity levels prescribed by the National Building Code (NBCC 1990) for the tower site. This is certainly an improvement over the past edition, which did not address the subject, but we have to recognize that tower designers need rational simplified methods as an alternative to detailed seismic analyses. This need was the main motivation for this study. In fact, we have realized that without adequate guidance, tower designers might be tempted to use the NBCC static approach, which is not valid for towers.

Our goal in this project was therefore to develop a static method in which the horizontal inertia effects due to base motion would be modelled by equivalent mass-proportional lateral loads. The main difficulty was to define a lateral acceleration profile that would represent realistic envelopes of tower accelerations in response to earthquakes. On the one hand, we wanted to keep the method fully static so that the designers would not even have to run a frequency analysis of the tower in order to find the

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natural frequencies and mode shapes of the tower. On the other hand, we wanted the method to be general enough so that more detailed data on these eigenvalues and eigenvectors could be used. The method would also have to be based on rational dynamic analysis principles of modal superposition in systems with distributed mass and stiffness. The linear theory of transverse vibrations of cantilever beams is the basis of the proposed method and it is reviewed in the next section.

TRANSVERSE VIBRATIONS OF ELASTIC CANTILEVER BEAMS

Eigensolution

Self-supporting towers behave essentially as cantilever beams, which have been amply studied for different cases of cross-sectional properties, including taper ratio and transverse shear deformation effects. Closed form expressions for the transverse vibration mode shapes and natural frequencies are available in the literature only for prismatic beams of constant cross section. The effect of taper ratio (Housner and Keightley 1962) and transverse shear deformations (Kruszewski 1949 and Cowper 1966) is to reduce the higher frequencies of the structure when compared to the classical prismatic beam.

The natural frequencies of a tapered cantilever in transverse vibration can be expressed as:

$$f_i = \frac{\lambda_i^2}{2\pi L^2}\sqrt{\frac{EI_0}{m_0}} \tag{1}$$

where EI_0 is the flexural rigidity at the base (fixed end), L is the total length, and m_0 is the mass per unit length at the base, and

$$\lambda_i = \lambda_B\, F_{ct}\, F_{cs}. \tag{2}$$

λ_B is the frequency parameter corresponding to pure bending in a prismatic cantilever, and factors F_{ct} and F_{cs} are reduction factors that modify the classical solution to account for the effect of taper and transverse shear deformations, respectively. The correction for taper effect, F_{ct}, is not very sensitive in the range of ratios D_0/D_L larger than 4, where D_0 and D_L are the member dimensions at the base (fixed end) and at the top (free end). Values of D_0/D_L for self-supporting latticed telecommunication towers are usually above 4. The correction for transverse shear deformations, F_{cs}, is more dependent on tower geometry. It would be feasible to develop a combined model for the variation of F_{ct} and F_{cs} in the specific range of taper ratios and lattice patterns used in self-supporting towers. Such a model could account for particularities of cross-sectional shapes (triangular or rectangular), types of leg members (open shapes or tubular members), and lateral bracing patterns. In the absence of such a model, it is possible to use published theoretical results for the tapered rectangular section (Housner and Keightley 1962, Downs 1977, and Blevins 1979).

The mode shapes corresponding to the natural frequencies of a prismatic cantilever beam are:

$$\phi_i(x) = \cosh(\lambda_i\, x/L) - \cos(\lambda_i\, x/L) - \sigma_i[\sinh(\lambda_i\, x/L) - \sin(\lambda_i\, x/L)] \tag{3}$$

where $\sigma_i = \dfrac{\sinh\lambda_i - \sin\lambda_i}{\cosh\lambda_i + \cos\lambda_i}$, and the origin of coordinate x is taken at the base (fixed end). Note that in the proposed static method only the lowest three mode shapes ($i=1,2,3$) will be used in modal superposition to determine the amplitudes of the accelerations induced in the tower when subjected to a harmonic pulse-type horizontal acceleration.

Undamped response to harmonic lateral base acceleration

Our proposed method requires the determination of a lateral acceleration profile appropriate to the tower site, considering the essential dynamic characteristics of the tower. Using the elastic cantilever beam model described above, we propose to determine the amplitude of the acceleration of the tower based on the undamped response to a harmonic base excitation $\ddot{u}_g \sin \Omega t$.

We know from previous studies (Mikus 1994) that telecommunication towers respond mainly in the lowest three flexural modes when subjected to horizontal base accelerations. This type of response, however, is expected only when a symmetric distribution of mass is assumed. This assumption is generally satisfactory since transmission lines, ladders and other attachments are usually of a distributed nature and/or have a small mass compared to that of the tower itself. We will therefore consider a truncated modal superposition solution including only the lowest three transverse modes of the cantilever beam:

$$\ddot{u}(x,t) = \ddot{u}_g \{ \phi_1(x)\psi(\lambda_1)\bar{a}(\Omega,\omega_1,t) + \phi_2(x)\psi(\lambda_2)\bar{a}(\Omega,\omega_2,t) + \phi_3(x)\psi(\lambda_3)\bar{a}(\Omega,\omega_3,t) \} \quad (4)$$

where $\psi(\lambda_i) = 2\, \sigma_i/\lambda_i$ and $\omega_i = 2\pi f_i$.

At resonance in a given mode i, $\Omega = \omega_i$,

$$\bar{a}(\Omega,\omega_i,t) = 1/2\ (\sin\omega_i t + \omega_i t \cos\omega_i t). \quad (5)$$

For $\Omega \neq \omega_i$, and using the frequency ratio $r_i = \Omega/\omega_i$,

$$\bar{a}(\Omega,\omega_i,t) = [r_i/(1-r_i^2)]\ (\sin\omega_i t - r_i \sin\Omega t). \quad (6)$$

In order to find the most probable estimate of the response at any time t and coordinate x, the acceleration can be calculated using the SRSS values obtained from Eq.(4) and assuming resonance in the first three modes, i.e. one can compute $\ddot{u}_1 = |\ddot{u}|_{\Omega=\omega 1}$, $\ddot{u}_2 = |\ddot{u}|_{\Omega=\omega 2}$ and $\ddot{u}_3 = |\ddot{u}|_{\Omega=\omega 3}$ and then take $\ddot{u} = (\ddot{u}_1^2 + \ddot{u}_2^2 + \ddot{u}_3^2)^{1/2}$. Since resonance is assumed, this value of $\ddot{u}$ grows unbounded with time if the excitation is sustained. However, instead of a truly harmonic excitation, a harmonic pulse-type excitation is considered for a duration of one cycle of the resonant frequency, that is $2\pi/\Omega$. In that case, the response described above for the harmonic excitation remains valid for time $t \leq 2\pi/\Omega$.

PROPOSED STATIC METHOD

Outline of procedure

Firstly, it is assumed that a detailed three-dimensional model of the stiffness of the tower is available. This should not pose a problem since tower designers routinely create such a model for the regular static analysis/design cycles. Our proposed method is summarized in the four following steps.

1) Determination of the bilinear acceleration profile — $\ddot{u}(x)$ — appropriate to the tower site;
2) Distribution of the tower mass at tributary joints on the leg members — m_j at node j;
3) Calculation of the equivalent lateral forces by simple product — $m_j\ \ddot{u}(x_j)$;
4) Static analysis of the detailed model of the tower using the nodal forces determined in 3).

Steps 2 to 4 are self-explanatory, but Step 1 requires more discussion.

Determination of the acceleration profile

Although it is possible to determine a complete acceleration profile using the truncated modal superposition described in the previous section, we feel that this complicates the calculation procedure to a level that would not be acceptable to designers, unless a special software is available. Furthermore, our numerical studies have shown that such an acceleration profile does not necessarily provide more accuracy than simpler shapes. After experimenting with several shapes of acceleration profiles, it was found that a bilinear curve was satisfactory for accelerograms with low and medium A/V ratios as defined by Tso *et al.* (1992), i.e. for records having A/V $\leq$ 1.2 g/m/s where g is the gravitational acceleration. This bilinear curve is defined using the truncated modal superposition procedure to calculate the magnitude of the acceleration at the top of the tower and at a reference elevation near the top of the tower, where the change in slope is prescribed. A value of 5L/6 is used in the example presented in the figures (75 m on Fig. 2) but more calibration with different towers subjected to several representative accelerograms is needed before a final recommendation can be made. Work done so far hints that this height corresponds to the location of the first inflection point from the top in the third transverse mode of the tower. It is also influenced by the A/V ratio of the input accelerogram, which necessarily has an effect on the participation factor of the third flexural mode in the overall response.

NIKAMO TOWER EXAMPLE

We now present an application of the proposed method to a 90-m three-legged tower owned by Hydro-Québec and located in the James Bay area. The geometry of the tower is illustrated in Fig. 1 together with the most important mode shapes obtained from the eigensolution of a detailed three-dimensional model of the tower. The table provided at the left of the tower outline designates member groups and will be useful when results of member forces are discussed. The mode shapes are for the tower alone, without any antenna attached to it. The lowest three flexural modes are of immediate interest, with corresponding natural frequencies of 1.3 Hz, 3.9 Hz, and 7.0 Hz, and the lowest two torsional modes at frequencies of 3.6 Hz and 6.7 Hz are also shown for completeness. These torsional modes are not excited by horizontal accelerograms on the tower without its antennae. In reality, however, heavy antennae or antenna clusters attached on the tower faces at various discrete locations induce eccentric lateral inertia forces on the tower. These eccentric forces will increase the participation of the tower's lowest torsional modes, and possibly biaxial bending modes. This apparent complex behaviour of real towers is not an obstacle to the derivation of a simplified method based on transverse modes only. We simply recommend that eccentric lateral forces be modelled on the same basis as the other lateral forces, i.e. that the localized mass is multiplied by the corresponding value of the acceleration profile, resulting in a static torque at the antenna locations.

Fig. 2 shows the proposed bilinear acceleration profile (in dashed lines) in relation to bounds obtained from dynamic analysis of the tower subjected to 15 accelerograms belonging to the medium A/V class (Tso *et al.* 1992), and assuming 3% viscous damping for each mode. As discussed above,

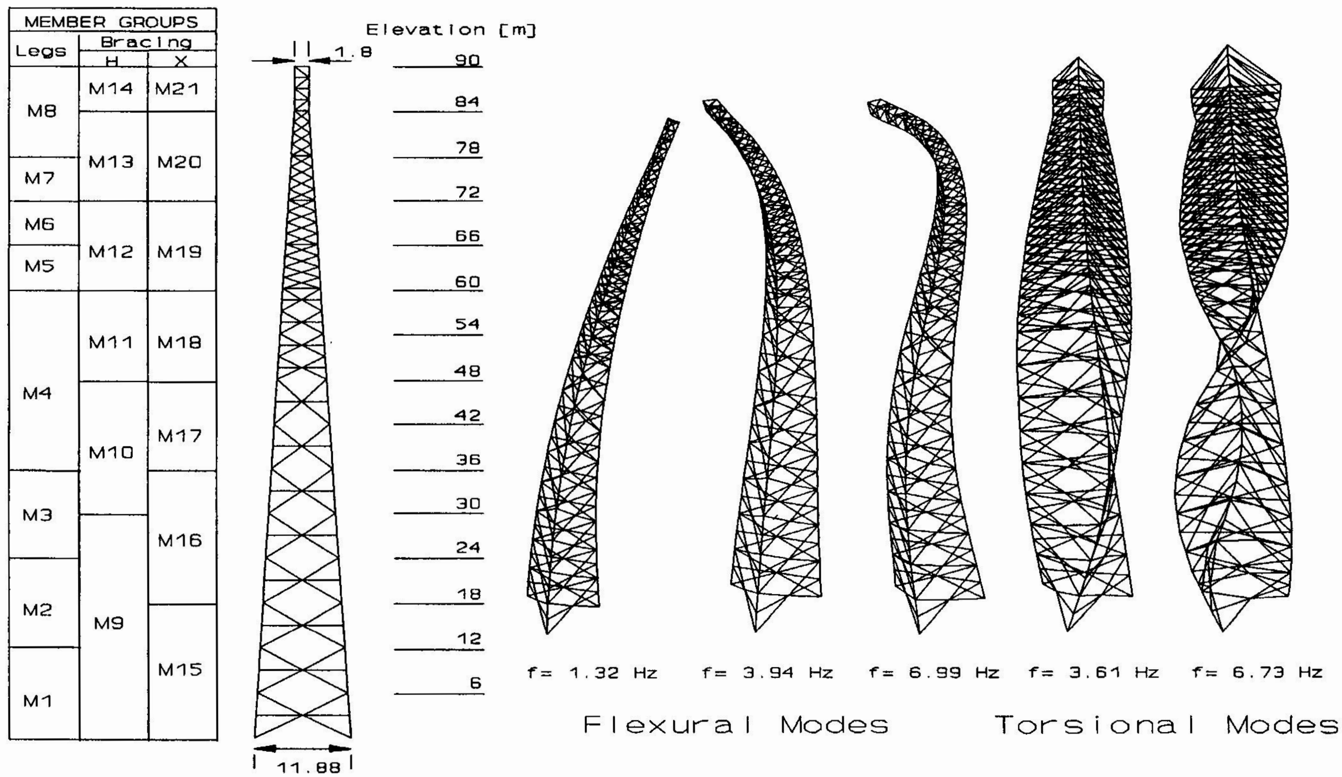

Fig. 1. Lowest natural frequencies and mode shapes of the Nikamo telecommunication tower

the maximum acceleration at the top of the tower was evaluated using Eqs. (4), (5) and (6) at time $t = 2\pi/\Omega$. Since the eigensolution had been carried out for the tower, we have used the numerical values of frequencies given in Fig. 1 instead of approximate values based on Eq. (2). The final result is $\ddot{u}_L = 8.4\ \ddot{u}_g$ and $\ddot{u}_{5L/6} = 3.0\ \ddot{u}_g$.

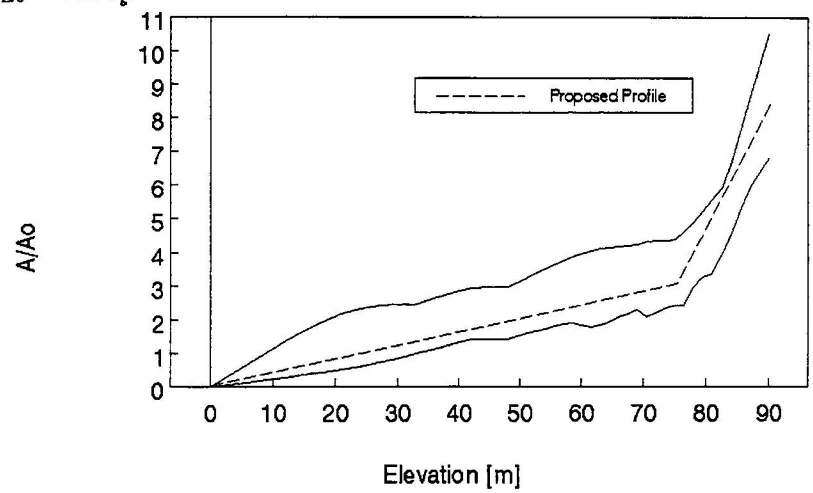

Fig. 2. Envelope of horizontal acceleration profiles obtained from dynamic analysis

Fig. 3 a) shows the absolute acceleration profile for $\ddot{u}_g = 0.28$ g. Note that this peak ground acceleration is not representative of the seismicity of the tower site, but it was used to give an indication of the order of magnitude of the maximum dynamic effects likely to occur on Canadian territory. We recommend that site-specific values of the peak ground acceleration given in the NBCC be used in the normal design process, however. Fig. 3 b) shows the mass distribution on each of the three tower legs, and Fig. 3 c) shows the equivalent lateral load on each leg, in the assumed direction of the earthquake. The three graphs of this figure are in fact the results obtained from steps 1, 2 and 3 of the proposed static method.

Analysis results are shown in Fig. 4 for leg member forces. Results obtained from the static method are compared with those of dynamic analysis with two accelerograms representative of the upper and lower bounds of the medium A/V class, as shown in Fig. 2. We are encouraged by the performance of our proposed method for all groups of members in this case (horizontal and vertical bracing results are not shown here). For leg members in particular, Fig. 4 indicates that the static method provides an envelope that differs on average by about 7% from the upper bound results of dynamic analysis.

We have studied the response of this tower to a total of 45 earthquake records but results for the low and high A/V classes are not shown due to space limitations. In summary, the same bilinear profile as for the medium A/V class fits well in the low A/V range also. However, a bilinear profile, overly conservative, is judged not satisfactory for high A/V (> 1.2 g/m/s) records. We are experimenting with different approaches in this case and should report our findings at a later date.

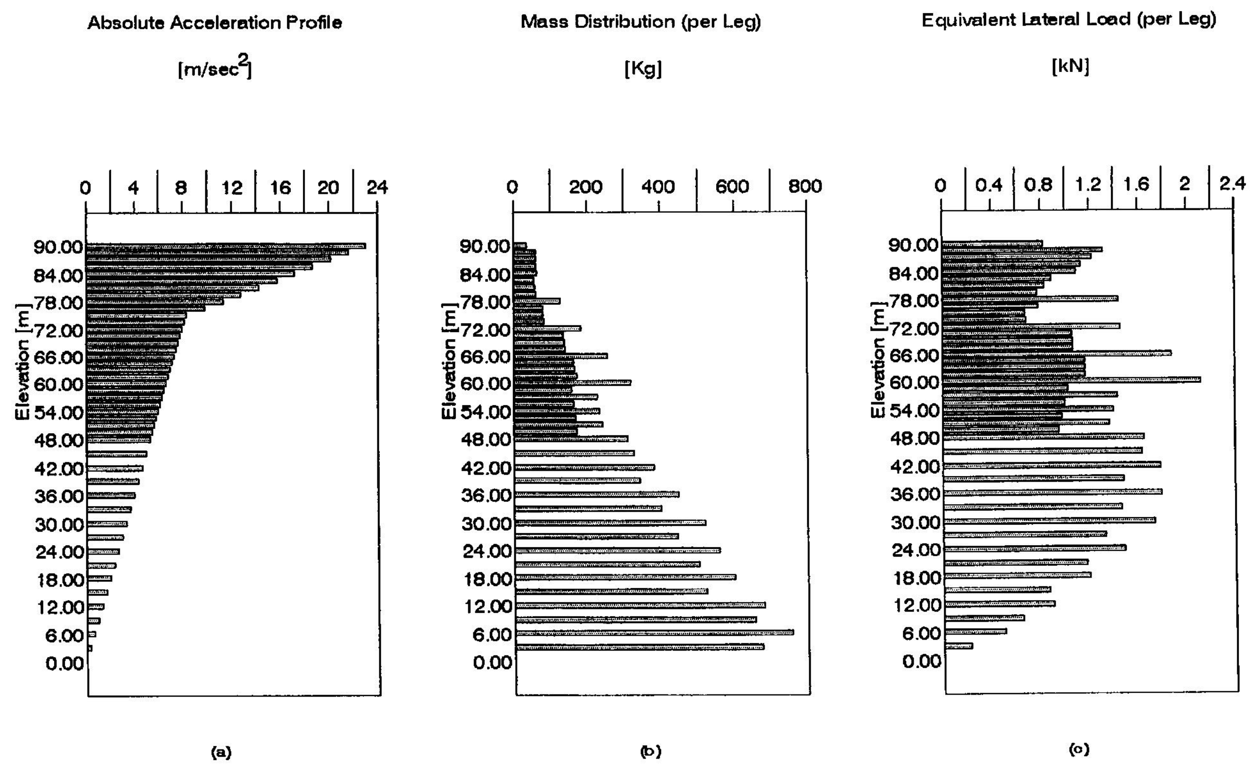

Fig.3. Equivalent lateral load obtained from the proposed bilinear acceleration profile

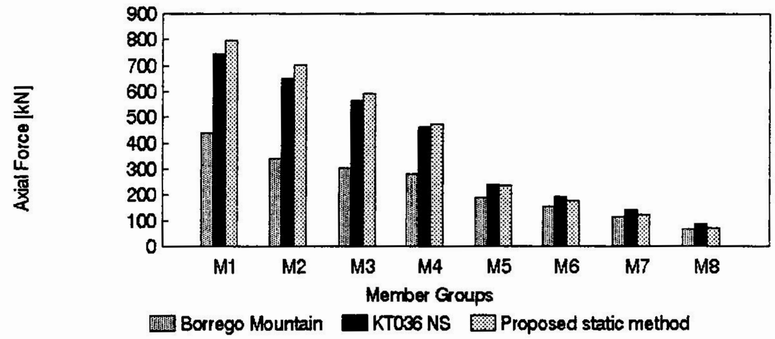

Fig. 4. Comparison between leg member forces obtained from the proposed static method and from dynamic analysis

CONCLUSIONS

We have presented an equivalent static method for the seismic analysis of self-supporting latticed towers. The results obtained for the detailed example presented show that the approach is feasible. The method still requires extensive calibration with more accelerograms and towers before presentation to the tower design community.

Acknowledgements

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